Jackson 11.3

Apply 2 successive transformations : $\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \to \begin{pmatrix} x_0' \\ x_1' \end{pmatrix} \to \begin{pmatrix} x_0'' \\ x_1'' \end{pmatrix}$ :

$$x_0' = \gamma'(x_0 - \beta' x_1) \qquad x_0'' = \gamma''(x_0' - \beta'' x_1')$$
$$x_1' = \gamma'(x_1 - \beta' x_0) \qquad x_1'' = \gamma''(x_1' - \beta'' x_0')$$

$$x_0'' = \gamma''(\gamma'(x_0 - \beta' x_1) - \beta'' \gamma'(x_1 - \beta' x_0))$$
$$= \gamma''\gamma'[(x_0 - \beta' x_1) - \beta''(x_1 - \beta' x_0)]$$
$$= \gamma''\gamma'[x_0 - \beta' x_1 - \beta'' x_1 + \beta''\beta' x_0]$$
$$= \gamma''\gamma'[(1+\beta''\beta')x_0 - (\beta'' + \beta')x_1]$$
$$= \gamma''\gamma'(1+\beta''\beta')\left[x_0 - \frac{(\beta''+\beta')}{(1+\beta''\beta')}x_1\right]$$

Consider $\gamma''\gamma'(1+\beta''\beta') = [1-\beta''^2]^{-1/2}[1-\beta'^2]^{-1/2}(1+\beta''\beta')$

$$= \left[\frac{(1+\beta''\beta')(1+\beta''\beta')}{(1-\beta''^2)(1-\beta'^2)}\right]^{1/2}$$

$$= \left[\frac{(1-\beta''^2)(1-\beta'^2)}{(1+\beta''\beta')(1+\beta''\beta')}\right]^{-1/2}$$

$$= \left[\frac{1+\beta''^2\beta'^2 - \cancel{2\beta''\beta'}\ \beta'^2 - \beta''^2}{1+\beta''^2\beta'^2 + 2\beta''\beta'}\right]^{-1/2}$$

$$= \left[ \frac{\left(1+\beta''^2\beta'^2+2\beta''\beta'\right) - 2\beta''\beta' - \beta'^2 - \beta''^2}{\left(1+\beta''^2\beta'^2+2\beta''\beta'\right)} \right]^{-1/2}$$

$$= \left[ 1 - \frac{\beta'^2+\beta''^2+2\beta''\beta'}{1+\beta''^2\beta'^2+2\beta''\beta'} \right]^{-1/2}$$

$$= \left[ 1 - \frac{(\beta'+\beta'')^2}{(1+\beta''\beta')^2} \right]^{-1/2}$$

$$= \boxed{\gamma(\beta) \quad \text{for} \quad \beta = \frac{\beta'+\beta''}{1+\beta''\beta'}}$$

$$\Rightarrow \boxed{\begin{gathered} x_0'' = \gamma\left[x_0 - \beta\, x_1\right] \\ \text{for} \quad \beta = \frac{\beta'+\beta''}{1+\beta''\beta'} \end{gathered}}$$

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