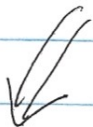


Jackson 11.3

Apply 2 successive transformations: $\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \rightarrow \begin{pmatrix} x_0' \\ x_1' \end{pmatrix} \rightarrow \begin{pmatrix} x_0'' \\ x_1'' \end{pmatrix}$:

$$\begin{aligned} x_0' &= \gamma'(x_0 - \beta'x_1) & x_0'' &= \gamma''(x_0' - \beta''x_1') \\ x_1' &= \gamma'(x_1 - \beta'x_0) & x_1'' &= \gamma''(x_1' - \beta''x_0') \end{aligned}$$


$$\begin{aligned} x_0'' &= \gamma''(\gamma'(x_0 - \beta'x_1) - \beta''\gamma'(x_1 - \beta'x_0)) \\ &= \gamma''\gamma'[(x_0 - \beta'x_1) - \beta''(x_1 - \beta'x_0)] \end{aligned}$$

$$= \gamma''\gamma' [x_0 - \beta'x_1 - \beta''x_1 + \beta''\beta'x_0]$$

$$= \gamma''\gamma' [(1 + \beta''\beta')x_0 - (\beta'' + \beta')x_1]$$

$$= \gamma''\gamma'(1 + \beta''\beta') \left[x_0 - \frac{(\beta'' + \beta')}{(1 + \beta''\beta')} x_1 \right]$$

Consider $\gamma''\gamma'(1 + \beta''\beta') = [1 - \beta''^2]^{-1/2} [1 - \beta'^2]^{-1/2} (1 + \beta''\beta')$

$$= \left[\frac{(1 + \beta''\beta')(1 + \beta''\beta')}{(1 - \beta''^2)(1 - \beta'^2)} \right]^{1/2}$$

$$= \left[\frac{(1 - \beta''^2)(1 - \beta'^2)}{(1 + \beta''\beta')(1 + \beta''\beta')} \right]^{-1/2}$$

$$= \left[\frac{1 + \beta''^2\beta'^2 - \cancel{2\beta''\beta'}\beta'^2 - \beta''^2}{1 + \beta''\beta' + 2\beta''\beta'} \right]^{-1/2}$$

$$= \left[\frac{(1 + \beta''^2 \beta'^2 + 2\beta''\beta') - 2\beta''\beta' - \beta'^2 - \beta''^2}{(1 + \beta''^2 \beta'^2 + 2\beta''\beta')} \right]^{-1/2}$$

$$= \left[1 - \frac{\beta'^2 + \beta''^2 + 2\beta''\beta'}{1 + \beta''^2 \beta'^2 + 2\beta''\beta'} \right]^{-1/2}$$

$$= \left[1 - \frac{(\beta' + \beta'')^2}{(1 + \beta''\beta')^2} \right]^{-1/2}$$

$$= \delta(\beta) \quad \text{for} \quad \beta = \frac{\beta' + \beta''}{1 + \beta''\beta'}$$

$$\Rightarrow \left[\begin{array}{l} x_0'' = \delta [x_0 - \beta x_1] \\ \text{for } \beta = \frac{\beta' + \beta''}{1 + \beta''\beta'} \end{array} \right]$$